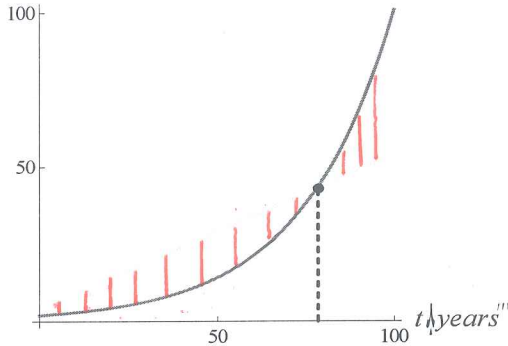


## Sec. 10.3 Combinations of Functions

The Difference of Two Functions Defined by Formulas: A Measure of Prosperity

Consider the population function  $P(t) = 2(1.04)^t$  and the number of people that a country can feed  $N(t) = 4 + 0.5t$  where  $t$  is measured in years and both  $P$  and  $N$  represent millions of people. When would the country first experience shortages?

millions of people

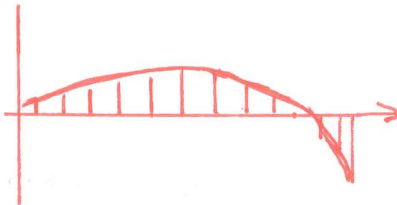


78.32 years (43.16 million people)

Now, how would you find the maximum surplus (ie when it is most prosperous)?

HINT: Need to find the difference between what? What should be higher?

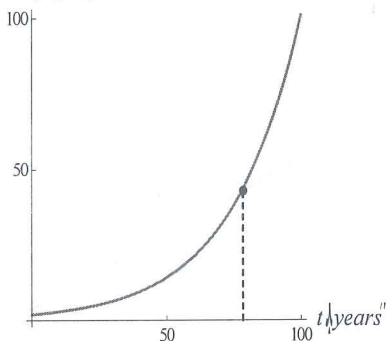
- Write an equation.  $S(t) = N(t) - P(t) = 4 + 0.5t - 2(1.04)^t$
- Graph the equation.
- Find the maximum surplus. 47.23 years - 14.86 million people



### The Sum and Difference of Two Functions defined by Graphs

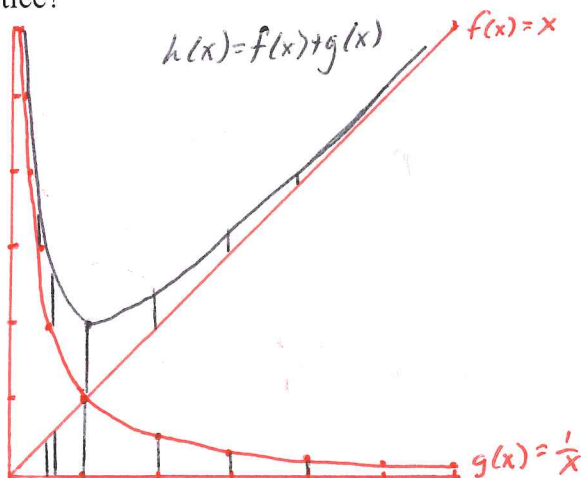
The surplus would be shown in between the two graphs which would then change to a shortage when the graphs switch locations.

millions of people



### Example 1

Let  $f(x) = x$  and  $g(x) = 1/x$ . By adding vertical distances on the graphs of  $f$  and  $g$ , sketch  $h(x) = f(x) + g(x)$  for  $x > 0$ . (Also sketch the other graphs as well on the same axis. What do you notice?)



Ex: Find exactly all the zeros of the function  $p(x) = 2^x \cdot 6x^2 - 2^x \cdot x - 2^{x+1}$ .

$$\begin{aligned}
 p(x) &= 2^x \cdot 6x^2 - 2^x \cdot x - 2^{x+1} \\
 0 &= 2^x \cdot 6x^2 - 2^x \cdot x - 2^x \cdot 2 \\
 0 &= 2^x (6x^2 - x - 2) \\
 0 &= 2^x (3x - 2)(2x + 1) \\
 2^x = 0 & \quad 3x - 2 = 0 & \quad 2x + 1 = 0 \\
 \text{NOT POSSIBLE} & \quad 3x = 2 & \quad 2x = -1 \\
 \text{(NO SOLUTION)} & \quad \boxed{x = \frac{2}{3}} & \quad \boxed{x = -\frac{1}{2}}
 \end{aligned}$$

Ex: Let  $u(x) = e^x$  and  $v(x) = 2x + 1$ . Solve:

A)  $f(x) = u(x)v(x)$

$$\begin{aligned}
 &e^x(2x+1) \\
 &2xe^x + e^x
 \end{aligned}$$

B.)  $f(x) = u(x)^2 v(x)^2$

$$\begin{aligned}
 &e^{x^2}(2x^2+1)^2 \\
 &2x^2e^{x^2} + e^{x^2}
 \end{aligned}$$

C.)  $f(x) = u((v(x))^2)$

$$\begin{aligned}
 (v(x))^2 &= (2x+1)^2 \\
 &= 4x^2 + 4x + 1
 \end{aligned}$$

$$u((v(x))^2) = e^{4x^2 + 4x + 1}$$

HW: pg 420-423 #6,7,9,11,15,20-23,24\*,26,27,30,31,36,37,40

\*Do parts c-e and you don't need to evaluate for  $x = 3$ .

Use graph paper for 31-40 and take make sure you are able to construct the graphs by hand.